THE ECONOMETRIC MODELLING OF COPULAS: A REVIEW WITH EXTENSIONS

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ABSTRACT. Copulas have been widely used in the recent years to build flexible joint risk distribution. They allow for a rich dependence structure and more realistic marginal distributions that better fit the features of empirical data, such as skewness and fat tails. Different estimation methods have been proposed, from parametric to non-parametrics ones. We review the methodologies suggested so far, providing the asymptotic distribution for each of them with some extensions.

1 INTRODUCTION

Evidence that many common economic variables are non-normal has been widely reported, as far back as Mills (1927). Common examples of deviations from univariate normality are excess kurtosis and skewness in univariate distributions. Recent studies of equity returns have also reported deviations from multivariate normality, in the form of asymmetric dependence .

The implication of these papers is that the multivariate normal distribution is simply not a good model for the joint distribution of many economic and financial variables. This leads us to the problem of finding more appropriate multivariate models. Copula theory can be a solution to this problem.

The theory of copulas dates back to Sklar(1959), but its application in statistical modelling is far more recent and dates back to the late '90, instead.

A copula is a function that embodies all the information about the dependence structure between the components of a random vector. When it is applied to marginal distributions which do not necessarily belong to the same distribution family, it results in a proper multivariate distribution. As a consequence, this theory enables us to incorporate a flexible modelling of the dependence structure between different variables, while allowing them to be modelled by different marginal distributions.

Nelsen (1999) provide an introduction to copula theory, while Cherubini et al.(2004) provide a discussion of copula techniques for financial applications.

Different estimation methods have been proposed in the recent years, ranging from parametric, till semi-parametric ones, up to non-parametric ones. Moreover, recent research tend to "mix" these methods to save computational time (Bouye' et al. 2000, Marshal and Zeevi 2002, Cherubini et al. 2004). However, the asymptotic properties of these methods have not been studied in all cases (particularly, the recent semiparametric ones).

What we do in this work is to review all these estimation methods, providing the asymptotic distribution for each of them, and propose some extensions.

The rest of the paper is organized as follows. In Section 2 we provide an outline of copula theory while in Section 3 we present the parametric models used for estimating copulas. We show in Section 4 the semi-parametric ones, while we conclude in Section 5.